

The NLO jet vertex for Mueller-Navelet and forward jets: the gluon part

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Abstract. In this paper we complete our calculation of the NLO jet vertex which is part of the cross section formulae for the production of Mueller Navelet jets at hadron hadron colliders and of forward jets in deep inelastic electron proton scattering.

1 Introduction

In a recent paper [1], we have started the NLO calculation of the jet vertex which represents one of the building blocks in the production of Mueller-Navelet jets [2] at hadron hadron colliders and of forward jets [3] in deep inelastic electron proton scattering. Both jet production processes are of particular interest for studying QCD in the Regge limit or in the small- x limit: they provide kinematical environments for which the BFKL Pomeron [4] applies. Previous experience shows that existing leading order calculations [5–7] are not accurate enough to allow for a reliable comparison with experimental data. NLO calculations are available for the BFKL Pomeron [8,9], but consistent next-to-leading order analysis of data at the Tevatron, at LHC, or at HERA require the NLO calculations also of the jet vertex and of the photon impact factor. As to the jet vertex, in our previous paper [1] we have presented the first part, namely the quark-initiated jet vertex. In the present paper, we complete the NLO analysis of the jet vertex with the gluon-initiated part. The NLO calculation of the photon impact factor is being pursued by two independent groups [10,11]. As a result of these combined efforts, it will be possible to perform consistent NLO studies of BFKL predictions in hadron hadron colliders, in ep deep inelastic scattering, and in e^+e^- scattering processes.

A particular theoretical challenge in computing the jet vertex for the processes mentioned before is related to the special kinematics. The processes to be analyzed is illustrated in Fig. 1: the lower gluon emitted from the hadron H scatters with the upper parton q and produces the jet J . Because of the large transverse momentum of the jet,

the gluon is hard and obeys the collinear factorization. In particular, its scale dependence is described by the DGLAP evolution equations [12]. Above the jet, on the other hand, we require a large rapidity gap between the jet and the outgoing parton q ; this kinematic requirement is described by BFKL dynamics. Consequently, the jet vertex lies at the interface between DGLAP and BFKL dynamics. As an essential result of our analysis we find that it is possible to separate, inside the jet vertex, the collinear infrared divergences that go into the parton evolution of the incoming gluon from the high energy gluon radiation inside the rapidity gap which belongs to the first rung of the LO BFKL ladder. In [1] this was demonstrated for the quark-initiated vertex, and in the present paper we present the generalization to the gluon part.

Our paper will be organized as follows. In order to make the reading as convenient as possible, we first review the general framework, which will be the same as in our previous paper [1]. Sections 3 and 4 then contain the virtual and real corrections respectively, and in Sect. 5 we combine both results. Section 6 contains, as a consistency check, a comparison of our jet vertex with the gluon impact factor. In Sect. 7 we use our results to define the general NLO jet production cross section. The concluding section contains a short summary and an outlook of future steps.

2 High energy factorization

2.1 General framework

We describe the kinematics of the hadron (H) quark (q) collision in terms of light cone coordinates

$$p^\mu = (p^+, p^-, \mathbf{p}), \quad p^\pm := \frac{p^0 \pm p^3}{\sqrt{2}}, \quad (1)$$

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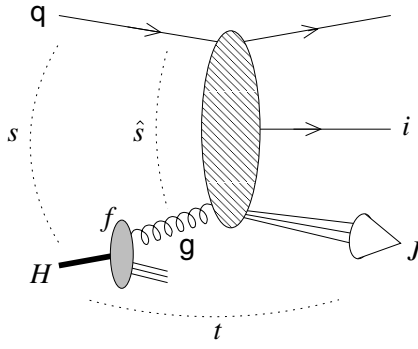


Fig. 1. Diagrammatic representation of the high energy process studied in this paper. H is the incoming hadron providing a gluon g with distribution density f ; q is the other incoming particle which will be taken to be a quark; J denotes the jet produced in the forward direction (w.r.t H) and i is the generic label for outgoing particles

where the light-like vectors p_H and p_q form the basis of the longitudinal plane:

$$p_H = \left(\sqrt{\frac{s}{2}}, 0, \mathbf{0} \right), \quad s := (p_H + p_q)^2 \quad (2a)$$

$$p_q = \left(0, \sqrt{\frac{s}{2}}, \mathbf{0} \right) \quad (2b)$$

$$p_i = E_i \left(\frac{e^{y_i}}{\sqrt{2}}, \frac{e^{-y_i}}{\sqrt{2}}, \phi_i \right). \quad (2c)$$

In the last equation we have introduced a parameterization for the i -th particle in the final state in terms of the rapidity y_i (in the $p_H + p_q$ center of mass frame), of the transverse energy $E_i = |\mathbf{p}_i|$ and of the azimuthal unit vector $\phi_i \parallel \mathbf{p}_i$.

According to the parton model, we assume the physical cross section to be given by the corresponding partonic cross section $d\hat{\sigma}$ (computable in perturbation theory) convoluted with the parton distribution densities (PDF) f_a of the partons a inside the hadron H . A jet distribution S_J selects the final states contributing to the one jet inclusive cross section that we are considering.

In terms of the jet variables – rapidity, transverse energy and azimuthal angle – the one jet inclusive cross section initiated by the gluons in hadron H can be written as

$$\frac{d\sigma}{dJ} := \frac{d\sigma_{qH}}{dy_J dE_J d\phi_J} = \int_0^1 dx d\hat{\sigma}_{qg}(x) S_J(x) f_g^{(0)}(x). \quad (3)$$

The incoming gluon carries a fraction x of the longitudinal momentum of H , while its transverse motion is negligible in the high energy regime:

$$p_g = x p_H = \left(x \sqrt{\frac{s}{2}}, 0, \mathbf{0} \right). \quad (4)$$

In our analysis we study the partonic subprocess $g + q \rightarrow X + jet$ in the high energy limit

$$\Lambda_{\text{QCD}}^2 \ll E_J^2 \sim -t \text{ (fixed)} \ll s \rightarrow \infty \quad (5)$$

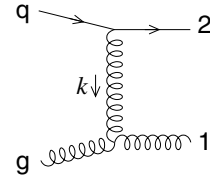


Fig. 2. Leading diagram at lowest order for gluon-quark scattering: the interaction occurs via gluon exchange in the t -channel

2.2 The jet vertex at lowest order

In the high energy regime (5), the lowest order (LO) contribution to the jet cross section is dominated by gluon exchange in the t -channel, as shown in Fig. 2.

The corresponding expression in $D = 4+2\epsilon$ dimensions has been already obtained in [1] and is given by

$$\frac{d\sigma^{(0)}}{dJ} = \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x) \quad (6)$$

in terms of the LO jet vertex

$$V_g^{(0)}(\mathbf{k}, x) := h_g^{(0)}(\mathbf{k}) \mathcal{S}_J^{(2)}(\mathbf{k}; x), \quad (7)$$

which is just the product of the LO gluon impact factor

$$h_g^{(0)}(\mathbf{k}) := \mathcal{N} \frac{C_A}{\mathbf{k}^2}, \quad \mathcal{N} = \frac{2^{1+\epsilon} \alpha_s}{\mu^{2\epsilon} \Gamma(1-\epsilon) \sqrt{N_c^2 - 1}}, \quad (8)$$

and of the jet distribution for two particles in the final state

$$\begin{aligned} \mathcal{S}_J^{(2)}(\mathbf{k}; x) &:= S_J^{(2)}(p_1, p_2; p_g, p_q) \\ &= \delta \left(1 - \frac{x_J}{x} \right) E_J^{1+2\epsilon} \delta(\mathbf{k} - \mathbf{k}_J), \\ x_J &:= \frac{E_J e^{y_J}}{\sqrt{s}}. \end{aligned} \quad (9)$$

2.3 Ansatz for the factorization formula

According to the analysis of [1], we propose a high energy factorization formula for the description of the high energy quark-hadron interaction with a jet in the final state. In this formula a quark impact factor h_q , the Green's function G for gluon exchange in the singlet channel, the gluon-initiated jet vertex V_g and the gluon PDF f_g are convoluted in both transverse and longitudinal variables to provide the jet differential cross section as follows:

$$\frac{d\sigma}{dJ} = \int dx \int d\mathbf{k} d\mathbf{k}' h_q(\mathbf{k}) G(xs, \mathbf{k}, \mathbf{k}') V_g(\mathbf{k}', x) f_g(x) \quad (10a)$$

$$G(xs, \mathbf{k}, \mathbf{k}') := \delta(\mathbf{k} - \mathbf{k}') + \alpha_s K^{(0)}(\mathbf{k}, \mathbf{k}') \log \frac{xs}{s_0} + \mathcal{O}(\alpha_s^2). \quad (10b)$$

The Green's function contains by definition all the energy dependence of the process and is given in terms of the

BFKL kernel [4] $K(\mathbf{k}, \mathbf{k}')$. By writing the perturbative expansions for the quark impact factor, the jet vertex, and the PDF

$$h = h^{(0)} + \alpha_s h^{(1)} + \dots \quad (11a)$$

$$V = V^{(0)} + \alpha_s V^{(1)} + \dots \quad (11b)$$

$$f = f^{(0)} + \alpha_s f^{(1)} + \dots, \quad (11c)$$

our ansatz corresponds to the following structure for the one-loop cross section:

$$\begin{aligned} \frac{d\sigma^{(1)}}{dJ} = & \alpha_s \int dx \int d\mathbf{k} \left\{ \int d\mathbf{k}' \left[h_q^{(0)}(\mathbf{k}) K^{(0)}(\mathbf{k}, \mathbf{k}') \right. \right. \\ & \times \log \frac{xs}{s_0} V_g^{(0)}(\mathbf{k}', x) f_g^{(0)}(x) \left. \right. \\ & + h_q^{(1)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x) \\ & + h_q^{(0)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(1)}(x) \\ & \left. \left. + h_q^{(0)}(\mathbf{k}) V_g^{(1)}(\mathbf{k}, x) f_g^{(0)}(x) \right\}, \quad (12) \end{aligned}$$

which is obtained simply by expanding (10a) up to relative order α_s .

For the first order correction to the partonic impact factor, $h^{(1)}$, which appears on the second line, we can use the known expression of [13,14], and for the correction to the PDF, $f^{(1)}$, we have the usual convolution with the LO Altarelli-Parisi splitting functions:

$$\begin{aligned} \alpha_s f_a^{(1)}(x, \mu_F^2) & := \frac{\alpha_s}{2\pi} \frac{1}{\varepsilon} \left(\frac{\mu_F^2}{\mu^2} \right)^\varepsilon \sum_b \int_x^1 \frac{d\xi}{\xi} P_{ab}(\xi) f_b^{(0)} \left(\frac{x}{\xi} \right) \\ & = \frac{\alpha_s}{2\pi} \frac{1}{\varepsilon} \left(\frac{\mu_F^2}{\mu^2} \right)^\varepsilon \sum_b P_{ab} \otimes f_b^{(0)}. \quad (13) \end{aligned}$$

Because of the definition (14) for $\alpha_s = g^2 \mu^{2\varepsilon} [1 + \varepsilon(\gamma_E - \log 4\pi)]/4\pi$, (13) defines the one-loop PDF in the $\overline{\text{MS}}$ scheme. Finally, the correction term $V_g^{(1)}$ is what we want to compute in this paper.

Equations (10) and (11) constitute a highly non trivial ansatz, which will be shown to depend upon a careful separation of singular and finite pieces. Our main task will consist to identify the collinear singularities (13) to be absorbed in the parton densities, to check cancellation of the remaining infrared singularities, and, finally, to separate the terms proportional to $\log s$ which belong into the first line of (12). The remaining finite (in ε) and constant (in s) term will eventually be interpreted as one-loop correction to the jet vertex, $V_g^{(1)}$.

3 Virtual corrections

For the one-loop analysis of the gluon-initiated jet production process we adopt dimensional regularization in $D = 4 + 2\varepsilon$ dimensions and define, according to the $\overline{\text{MS}}$ scheme, the bare dimensionless coupling α_s as a function

of the dimensionful bare coupling g and of the renormalization scale μ as follows:

$$\alpha_s = \alpha_s^{(0)} := \frac{g^2 \mu^{2\varepsilon} \Gamma(1 - \varepsilon)}{(4\pi)^{1+\varepsilon}} \quad (14)$$

The one-loop analysis of the virtual corrections can be carried out in the same way of the quark-initiated case presented in [1]. Discarding all terms suppressed by powers of s , the one-loop quark-gluon cross section can be derived from [15] and the ensuing virtual contribution to the jet cross section reads

$$\begin{aligned} \frac{d\sigma^{(\text{virt})}}{dJ} = & \alpha_s \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \\ & \times \left[2\omega^{(1)}(\mathbf{k}) \log \frac{xs}{\mathbf{k}^2} + \tilde{\Pi}_q(\mathbf{k}) + \tilde{\Pi}_g(\mathbf{k}) \right] \\ & \times V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x), \quad (15) \end{aligned}$$

$h_q^{(0)}(\mathbf{k}) = h_g^{(0)}(\mathbf{k}) C_F/C_A$ being the LO quark impact factor. The first term represents the leading log s (LL) contribution to the virtual corrections. The coefficient of $\log s$, namely $2\omega^{(1)}$, constitutes the virtual part of the leading BFKL kernel and is just twice the one-loop Regge-gluon trajectory

$$\omega^{(1)}(\mathbf{k}) = -\frac{C_A}{\pi} \frac{1}{2\varepsilon} \frac{\Gamma^2(1 + \varepsilon)}{\Gamma(1 + 2\varepsilon)} \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\varepsilon. \quad (16)$$

The ε -pole reflects the presence of a soft singularity which will be compensated by an opposite one in the real part of the kernel.

The non logarithmic terms in (15) represent the next-to-leading log s (NLL) contribution to the virtual corrections after the subtraction of the UV ε -pole occurring in the renormalization of the coupling

$$\alpha_s(\mu^2) := \alpha_s^{(0)} \left[1 - \alpha_s^{(0)} \frac{b_0}{\varepsilon} \right]. \quad (17)$$

They are expressed in terms of the full virtual corrections to the impact factors [15] Π_a : $\mathbf{a} = \mathbf{q}, \mathbf{g}$, and are denoted by $\tilde{\Pi}_a := \Pi_a - (-b_0/\varepsilon)$. The explicit expressions for the quark impact factor correction were given in (40,45) of [1]; the virtual correction to the gluon impact factor is

$$\begin{aligned} \tilde{\Pi}_g(\mathbf{k}) = & \left[\left(-\frac{1}{\varepsilon^2} + \frac{11}{6\varepsilon} + \frac{5\pi^2}{12} - \frac{67}{36} \right) \frac{C_A}{\pi} \right. \\ & \left. + \left(-\frac{1}{3\varepsilon} + \frac{5}{18} \right) \frac{N_f}{\pi} - b_0 \log \frac{\mathbf{k}^2}{\mu^2} \right] \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\varepsilon, \quad (18) \end{aligned}$$

where $b_0 = (11C_A - 2N_f)/12\pi$ is the first coefficient of the β -function. Any occurrence of α_s in (15) and in all other coming formulae is to be understood as $\alpha_s(\mu^2)$.

The gluon impact factor virtual correction (18) shows double and single poles in ε . These poles are of IR origin and are due to both soft and collinear singularities. Partly they will cancel against the corresponding singularities of the real emission corrections, leaving a simple pole that will be absorbed in the redefinition of the PDFs. This will be shown in Sect. 5.

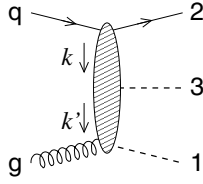


Fig. 3. Labelling the two-to-three parton scattering process. The two dashed lines denote a $q\bar{q}$ pair or a gluon pair. Note the definition of the transferred momenta k and k'

4 Real corrections

The real emission corrections of the quark-gluon collision that we are considering, involves three partons in the final states: an outgoing quark (labelled by “2”) – which is nothing but the scattered incoming quark q – and two additional partons (“1” and “3”), which can be either a $q\bar{q}$ pair or two gluons (see Fig. 3).

We parametrize the kinematics of the process in terms of Sudakov variables of two exchanged momenta:

$$k := p_q - p_2 = -\bar{w}p_g + wp_q + k_\perp, \quad k_\perp = (0, 0, \mathbf{k}) \quad (19a)$$

$$k' := p_1 - p_g = -zp_g + \bar{z}p_q + k'_\perp, \quad k'_\perp = (0, 0, \mathbf{k}'). \quad (19b)$$

Note that the transverse energies introduced in (2c) correspond to $E_1 = |\mathbf{k}'|$, $E_2 = |\mathbf{k}|$, $E_3 = |\mathbf{k} - \mathbf{k}'|$.

The partonic differential cross sections has been computed [13,14] for both cases in the high energy regime, where terms suppressed by powers of s are neglected. Introducing the rapidity $y' = y + \frac{1}{2} \log \frac{1}{x}$ “measured” in the partonic center of mass frame, we can split the phase space into two parts defined by $y'_1, y'_3 > 0$ (*lower half*) and $y'_3 < 0 < y'_1$ (*upper half*)¹.

The form of the partonic differential cross section turns out to be quite simple when restricted to one of the two halves of the phase space. For the “lower half region” $y'_1, y'_3 > 0$, which corresponds to $z > z_{\text{cut}} := \frac{E_3}{\sqrt{x s}}$, the cross section can be cast into the general form

$$d\hat{\sigma}_{qg \rightarrow fin} = h_q^{(0)}(\mathbf{k}) F_{fin}(\mathbf{k}, \mathbf{k}', z) h_g^{(0)}(\mathbf{k}') d\mathbf{k} d\mathbf{k}' dz, \quad (z > z_{\text{cut}}), \quad (20)$$

where the function F depends on the particular final state. For quark-gluon scattering, we have two contributions. The $qq\bar{q}$ final state term is given by

$$F_{qq\bar{q}}(\mathbf{k}, \mathbf{k}', z) = \frac{\alpha_s}{2\pi} N_f T_R \frac{\mathcal{P}_{qg}(z, \varepsilon)}{\pi_\varepsilon} \frac{1}{q^2} \times \left[\frac{C_F}{C_A} + z(1-z) \frac{\mathbf{q} \cdot \mathbf{k}'}{(\mathbf{q} - z\mathbf{k})^2} \right], \quad (21)$$

$$\mathcal{P}_{qg}(z, \varepsilon) = 1 - \frac{2z(1-z)}{1+\varepsilon}, \quad (22)$$

$$\pi_\varepsilon = \pi^{1+\varepsilon} \Gamma(1-\varepsilon) \mu^{2\varepsilon},$$

¹ By momentum conservation there is always a particle with $y' > 0$, which can be labelled by “1” without loss of generality

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, $T_R = 1/2$ and $\mathcal{P}_{qg}(z, \varepsilon)$ is – apart from a missing T_R factor – the real part of the $g \rightarrow q$ splitting function in $4 + 2\varepsilon$ dimensions.

The qgg contribution can be written as

$$F_{qgg}(\mathbf{k}, \mathbf{k}', z) = \frac{\alpha_s}{2\pi} C_A \frac{\mathcal{P}_{gg}(z)}{\pi_\varepsilon} \times \frac{z^2 \mathbf{k}'^2 + (1-z)^2 \mathbf{q}^2 - z(1-z) \mathbf{q} \cdot \mathbf{k}'}{q^2 (\mathbf{q} - z\mathbf{k})^2}, \quad (23)$$

$$\mathcal{P}_{gg}(z) = \frac{1 + z^4 + (1-z)^4}{2z(1-z)}, \quad (24)$$

where $\mathcal{P}_{gg}(z)$ is – apart from the missing colour factor $2C_A$ – the real part of the $g \rightarrow g$ splitting function (in any dimension).

The real correction to the upper quark impact factor receives contribution only from the qgg final state in the “upper half region” of the phase space and from the virtual correction contribution of (15) coming from the q impact factor correction \tilde{H}_q . It can be computed in a similar way as presented in [1] and it will not be discussed further here. We simply report the final result adapted to the case of incoming gluon:

$$\left. \frac{d\sigma_{qgg}^{(y'_3 < 0)}}{dJ} + \frac{d\sigma^{(\text{virt})}}{dJ} \right|_{\tilde{H}_q} = \alpha_s \int dx \int d\mathbf{k} d\mathbf{k}' h_q^{(0)}(\mathbf{k}) K^{(0, \text{real})}(\mathbf{k}, \mathbf{k}') \times \log \frac{\sqrt{x s}}{\max(E_2, E_3)} V_g^{(0)}(\mathbf{k}', x) f_g^{(0)}(x) + \alpha_s \int dx \int d\mathbf{k} h_q^{(1)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x), \quad (25)$$

The structure of the infrared (IR) singularities for the gluon-initiated vertex, is somewhat more entangled than that of the quark-initiated one. This happens in particular when the gluon pair is emitted in the final state. Since gluons are indistinguishable, each of the gluons can contribute to the LL term, and the LL subtraction needed to extract the jet vertex cannot be defined on the same footing as in the quark case [1].

In the next sections we develop an explicit procedure for separating the LL contribution from the collinear singular terms. We check the cancellation of the soft singularities and isolate the finite expression for the gluon-initiated jet vertex.

4.1 Jet definition

We begin with a brief review of the jet definition. Following the arguments given in [16], we require that the jet distribution $S_J^{(n)}$, selecting from a generic n -particle final state the configurations contributing to our one jet inclusive observable, be IR safe. This corresponds to the fact that emission of a soft particle cannot be distinguished

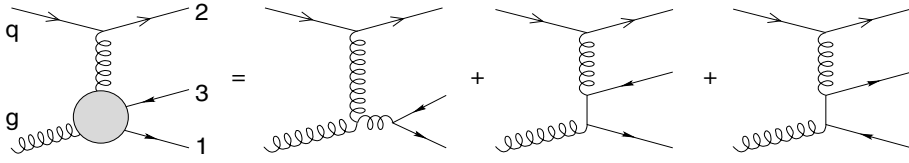


Fig. 4. Feynman diagrams contributing to quark-gluon scattering in which the gluon fragments in a $q\bar{q}$ pair

from the analogous state without soft emission. Furthermore, collinear emissions of partons cannot be distinguished from the corresponding state where the collinear partons are replaced by a single parton carrying the sum of their quantum numbers.

Listing the possible IR singular configurations, only the emission of a gluon (1 or 3) with vanishing momentum gives rise to soft singularities. Collinear singularities arise in collinear emissions of partons that couple directly to each other. The list of all possible collinear singular configurations, in an obvious notation, reads as follows:

$$g \parallel 1, \quad g \parallel 3, \quad 1 \parallel 3, \quad (26a)$$

$$q \parallel 1, \quad q \parallel 2, \quad q \parallel 3, \quad 2 \parallel 3. \quad (26b)$$

It is important to note that, in the kinematic regime we are considering, configurations in which quark 2 is emitted outside the fragmentation region of quark q are strongly suppressed, i.e., quark 2 never belongs to the jet produced in the forward direction of gluon g . This is because the propagator of the exchanged particle provides a suppression factor $\sim t/s$ with respect to the situation where quark 2 is in the fragmentation region of quark q . Therefore, we can safely neglect the configurations in which quark 2 enters the jet, and only particles 1 and 3 play a role in building up the jet.

In terms of the variables $\mathbf{p}_1, \mathbf{p}_3, p_3^+, p_g^+$, the 3-particle jet distribution

$$\begin{aligned} \mathcal{S}_J^{(3)}\left(\mathbf{p}_1, \mathbf{p}_3, \frac{p_3^+}{p_H^+}, \frac{p_g^+}{p_H^+}\right) &\equiv \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{k} - \mathbf{k}', xz; x) \\ &:= \mathcal{S}_J^{(3)}(p_1, p_2, p_3; p_g, p_q). \end{aligned} \quad (27)$$

must satisfy the following properties in order to be IR safe (cfr. [1]):

$$1 \text{ soft} : \quad \mathcal{S}_J^{(3)}(\mathbf{0}, \mathbf{p}, x; x) = \mathcal{S}_J^{(2)}(\mathbf{p}; x) \quad (28a)$$

$$3 \text{ soft} : \quad \mathcal{S}_J^{(3)}(\mathbf{p}, \mathbf{0}, 0; x) = \mathcal{S}_J^{(2)}(\mathbf{p}; x) \quad (28b)$$

$$1 \parallel 3 : \quad \mathcal{S}_J^{(3)}((1-\lambda)\mathbf{p}, \lambda\mathbf{p}, \lambda x; x) = \mathcal{S}_J^{(2)}(\mathbf{p}; x) \quad (28c)$$

$$g \parallel 1 : \quad \mathcal{S}_J^{(3)}(\mathbf{0}, \mathbf{p}, \xi; x) = \mathcal{S}_J^{(2)}(\mathbf{p}; \xi) \quad (28d)$$

$$g \parallel 3 : \quad \mathcal{S}_J^{(3)}(\mathbf{p}, \mathbf{0}, \xi; x) = \mathcal{S}_J^{(2)}(\mathbf{p}; x - \xi). \quad (28e)$$

Note that, since quark 2 does not participate in the jet, the collinear properties of the jet distribution are applied only to the configurations listed in (26a).

In the following sections these relations will be used when extracting the divergences of the real emission.

4.2 $q\bar{q}$ pair in the final state

We have already observed in the previous section that there are two kind of final states, differing in the type of the partons denoted by 1 and 3: either a $q\bar{q}$ pair or a gluon pair. Here we consider the contribution of the final state with $q\bar{q}$ pair. In this case, the main contribution occurs when the $q\bar{q}$ pair is emitted with a small invariant mass in the fragmentation region of the incoming gluon. In practice, we need only to consider the “lower half” phase space $y_3' < 0$ contribution. The corresponding Feynman diagrams are shown in Fig. 4.

The starting formula is derived from (3), using (27) for the jet distribution and (20) and (21) for the partonic cross section:

$$\begin{aligned} \frac{d\sigma_{q\bar{q}\bar{q}}}{dJ} &= \frac{\alpha_s}{2\pi} \mathcal{N} \int d\mathbf{k} d\mathbf{k}' h_q^{(0)}(\mathbf{k}) \\ &\times \int_{z_{\text{cut}}}^1 dz N_f T_R \frac{\mathcal{P}_{qg}(z, \varepsilon)}{\pi_\varepsilon} \frac{1}{\mathbf{k}'^2 \mathbf{q}^2} \\ &\times \left[C_F + C_A \frac{z(1-z)\mathbf{q} \cdot \mathbf{k}'}{(\mathbf{q} - z\mathbf{k})^2} \right] \\ &\times \int dx \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) f_g^{(0)}(x). \end{aligned} \quad (29)$$

Since the integrand in the RHS of (29) is regular in the $z \rightarrow 0$ limit, we can replace the lower limit of integration in z : $z_{\text{cut}} \rightarrow 0$. This amounts to a negligible error in the high energy limit of order t/s .

C_F term

Let us first consider the C_F piece. In this term the two IR singularities of collinear origin ($3 \parallel g \iff \mathbf{q} = 0$ or $1 \parallel g \iff \mathbf{k}' = 0$) can be disentangled employing a simple fract decomposition:

$$\frac{1}{\mathbf{q}^2 \mathbf{k}'^2} = \frac{1}{\mathbf{q}^2 + \mathbf{k}'^2} \left[\frac{1}{\mathbf{q}^2} + \frac{1}{\mathbf{k}'^2} \right], \quad (30)$$

and we can write

$$\frac{d\sigma_{C_F}}{dJ} = \frac{d\sigma_{C_F}^{(1)}}{dJ} + \frac{d\sigma_{C_F}^{(3)}}{dJ} \quad (31)$$

$$\frac{d\sigma_{C_F}^{(1)}}{dJ} = \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int \frac{d\mathbf{k}'}{\pi_\varepsilon \mathbf{k}'^2} I(\mathbf{k}'; \mathbf{k})$$

$$\frac{d\sigma_{C_F}^{(3)}}{dJ} = \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int \frac{d\mathbf{k}'}{\pi_\varepsilon (\mathbf{k} - \mathbf{k}')^2} I(\mathbf{k}'; \mathbf{k})$$

where the integrand I is given by

$$I(\mathbf{k}'; \mathbf{k}) := \mathcal{N} \frac{C_F T_R N_f}{2\pi} \frac{1}{\mathbf{k}'^2 + (\mathbf{k} - \mathbf{k}')^2} \int_0^1 dz \mathcal{P}_{\text{qg}}(z, \varepsilon) \times \int dx \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{k} - \mathbf{k}', xz; x) f_{\text{g}}^{(0)}(x). \quad (32)$$

Because of the symmetry of \mathcal{P}_{qg} and $\mathcal{S}_J^{(3)}$ under the $\mathbf{q} \leftrightarrow \bar{\mathbf{q}}$ exchange, i.e., $\{z \leftrightarrow 1-z, \mathbf{k}' \leftrightarrow \mathbf{k} - \mathbf{k}'\}$, it holds $I(\mathbf{k}'; \mathbf{k}) = I(\mathbf{k} - \mathbf{k}'; \mathbf{k})$ and thus the two terms in (31) are equal.

The separation of the collinear singularity is performed by means of the subtraction method by decomposing

$$\frac{d\sigma}{dJ} C_F := \frac{d\sigma}{dJ} C_F^{\text{coll}} + \frac{d\sigma}{dJ} C_F^{\text{finite}}, \quad (33)$$

where the collinear singular term depends on an UV cut-off Λ and is defined by

$$\frac{d\sigma}{dJ} C_F^{\text{coll}} := 2 \int d\mathbf{k} h_{\text{q}}^{(0)}(\mathbf{k}) \int \frac{d\mathbf{k}'}{\pi_\varepsilon \mathbf{k}'^2} I(\mathbf{0}; \mathbf{k}) \Theta(\Lambda^2 - \mathbf{k}'^2). \quad (34)$$

In the collinear limit $\mathbf{k}' \rightarrow 0$, the jet distribution reduces to $\mathcal{S}_J^{(3)} \rightarrow \mathcal{S}_J^{(2)}(\mathbf{k}; x)$ (cfr. (28d,e)). Introducing the gluon-to-quark splitting function $P_{\text{qg}}(z) = T_R[z^2 + (1-z)^2]$ one can write

$$\begin{aligned} \frac{d\sigma}{dJ} C_F^{\text{coll}} &= \frac{\alpha_s}{\pi} N_f \int d\mathbf{k} h_{\text{q}}^{(0)}(\mathbf{k}) \int dx \\ &\times \left[\frac{1}{\varepsilon} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon \int_0^1 dz P_{\text{qg}}(z) V_{\text{q}}^{(0)}(\mathbf{k}, xz) f_{\text{g}}^{(0)}(x) \right. \\ &\left. + \int_0^1 dz z(1-z) V_{\text{q}}^{(0)}(\mathbf{k}, xz) f_{\text{g}}^{(0)}(x) \right]. \quad (35) \end{aligned}$$

The finite contribution of the C_F term can be computed at $\varepsilon = 0$ and reads

$$\begin{aligned} \frac{d\sigma}{dJ} C_F^{\text{finite}} &= \frac{\alpha_s}{\pi} N_f \int d\mathbf{k} h_{\text{q}}^{(0)}(\mathbf{k}) \int \frac{d\mathbf{k}'}{\pi} \int dx \int_0^1 dz P_{\text{qg}}(z) \\ &\times \left[h_{\text{q}}^{(0)}(\mathbf{k}') \frac{1}{\mathbf{q}^2 + \mathbf{k}'^2} \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) \right. \\ &\left. - \frac{1}{\mathbf{k}'^2} \Theta(\Lambda^2 - \mathbf{k}'^2) V_{\text{q}}^{(0)}(\mathbf{k}, xz) \right] f_{\text{g}}^{(0)}(x). \quad (36) \end{aligned}$$

C_A term

In the C_A term there is only one final state collinear singularity when $3 \parallel 1$, corresponding to $\mathbf{q} - z\mathbf{k} = 0$. Also in this case we separate the collinear singularity

$$\frac{d\sigma}{dJ} C_A := \frac{d\sigma}{dJ} C_A^{\text{coll}} + \frac{d\sigma}{dJ} C_A^{\text{finite}} \quad (37)$$

by means of the subtraction method: we define the collinear singular term as the residue of the $1/(\mathbf{q} - z\mathbf{k})^2$ pole, integrated in a circle of radius Λ centered in the singularity. For simplicity we use the same cut-off Λ introduced in the C_F term. In this case, the jet distribution can be

simplified thanks to (28c): $\mathcal{S}_J^{(3)} \rightarrow \mathcal{S}_J^{(2)}(\mathbf{k}; x)$ and the z integration can straightforwardly be performed, yielding

$$\begin{aligned} \frac{d\sigma}{dJ} C_A^{\text{coll}} &= \alpha_s \int d\mathbf{k} h_{\text{q}}^{(0)}(\mathbf{k}) \int dx V_{\text{g}}^{(0)}(\mathbf{k}, x) f_{\text{g}}^{(0)}(x) \\ &\times \frac{N_f}{\pi} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon \left(\frac{1}{6\varepsilon} + \frac{1}{12} \right) \quad (38) \end{aligned}$$

and

$$\begin{aligned} \frac{d\sigma}{dJ} C_A^{\text{finite}} &= \frac{\alpha_s}{2\pi} N_f \int d\mathbf{k} h_{\text{q}}^{(0)}(\mathbf{k}) \int dx f_{\text{g}}^{(0)}(x) \\ &\times \int_0^1 dz P_{\text{qg}}(z) \int \frac{d\mathbf{k}'}{\pi} \frac{\mathcal{N} C_A}{(\mathbf{q} - z\mathbf{k})^2} \\ &\times \left[z(1-z) \frac{\mathbf{q} \cdot \mathbf{k}'}{\mathbf{q}^2 \mathbf{k}'^2} \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) \right. \\ &\left. - \frac{1}{\mathbf{k}'^2} \Theta(\Lambda^2 - (\mathbf{q} - z\mathbf{k})^2) \mathcal{S}_J^{(2)}(\mathbf{k}; x) \right] \quad (39) \end{aligned}$$

4.3 Real corrections to the jet vertex from qgg final state

The starting formula can be derived from (3), using (27) for the jet distribution and (20) and (23) for the partonic cross section. A more convenient expression can be written if one notes that the splitting function for gluons can be decomposed as

$$P_{\text{gg}}(z) = P(z) + P(1-z), \quad P(z) = \left(\frac{1}{z} + \frac{z}{2} \right) (1-z). \quad (40)$$

Using the symmetry of the partonic cross section and of the jet distribution under the exchange of the two gluons: $\{z \leftrightarrow 1-z, \mathbf{k}' \leftrightarrow \mathbf{k} - \mathbf{k}'\}$, the jet cross section can be rewritten as

$$\begin{aligned} \frac{d\sigma}{dJ} C_A^{\text{qgg}} &= \frac{\alpha_s}{\pi} \int d\mathbf{k} h_{\text{q}}^{(0)}(\mathbf{k}') \int \frac{d\mathbf{k}'}{\pi_\varepsilon} h_{\text{g}}^{(0)}(\mathbf{k}) \int_{z_{\text{cut}}}^1 dz P(z) \\ &\times \frac{z^2 \mathbf{k}'^2 + (1-z)\mathbf{q} \cdot (\mathbf{q} - z\mathbf{k})}{\mathbf{q}^2 (\mathbf{q} - z\mathbf{k})^2} \\ &\times \int dx \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) f_{\text{g}}^{(0)}(x). \quad (41) \end{aligned}$$

It is important to observe that now the ‘‘splitting function’’ P has a pole only at $z = 0$, while it is regular (actually vanishes) at $z = 1$. The reason is that now we are employing an asymmetric treatment which causes gluon 3 to be the only responsible of soft singularities and of the central region LL contribution.

The expression (41) looks now pretty similar to the real contribution of the quark-initiated case. We can therefore proceed in an analogous way for its computation and consider separately the two terms in (41). We define the A and B term as the first ($\propto z^2 \mathbf{k}'^2$) and the second ($\propto (1-z)\mathbf{q} \cdot (\mathbf{q} - z\mathbf{k})$) terms of (41) respectively.

4.3.1 A term

The A term² numerator, has no $\mathbf{g} \parallel 1$ collinear singularity. In addition, the z^2 factor in the numerator causes a suppression of the integrand in the central region, and we can shift the lower limit of integration z_{cut} in z down to zero:

$$\begin{aligned} \frac{d\sigma^A}{dJ} &= \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int dx f_g^{(0)}(x) \int_0^1 dz z^2 P(z) \\ &\times \int \frac{d\mathbf{k}'}{\pi_\varepsilon} \frac{\mathcal{N}C_A}{q^2(\mathbf{q} - z\mathbf{k})^2} \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x). \end{aligned} \quad (42)$$

We perform the rescaling $\mathbf{q} =: z\mathbf{l}$ and use \mathbf{l} as integration variable by substituting $\mathbf{k}' = \mathbf{k} - z\mathbf{l}$, so that $\mathbf{q} - z\mathbf{k} = z(\mathbf{l} - \mathbf{k})$. Next we perform a simple fraction decomposition in order to separate the initial (i) state ($\mathbf{g} \parallel 3 \iff \mathbf{l} = 0$) and final (f) state ($1 \parallel 3 \iff \mathbf{l} - \mathbf{k} = 0$) collinear singularities:

$$\frac{1}{\mathbf{l}^2(\mathbf{l} - \mathbf{k})^2} = \frac{1}{\mathbf{l}^2 + (\mathbf{l} - \mathbf{k})^2} \left[\frac{1}{\mathbf{l}^2} + \frac{1}{(\mathbf{l} - \mathbf{k})^2} \right]. \quad (43)$$

Beginning with the **final state** (f) collinear singularity, in terms of the new variables the A contribution to the jet cross section can be rewritten in the form

$$\begin{aligned} \frac{d\sigma^f}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \int \frac{d\mathbf{l}}{\pi_\varepsilon(\mathbf{l} - \mathbf{k})^2} I(z, \mathbf{l}) \\ &= \frac{d\sigma_A^{f,\text{soft}}}{dJ} + \frac{d\sigma_A^{f,\text{coll}}}{dJ} + \frac{d\sigma_A^{f,\text{finite}}}{dJ}, \end{aligned} \quad (44)$$

which is particularly suitable for the analytic extraction of the divergences: the RHS contains three pieces, 1) the soft divergence, 2) the collinear divergence, and 3) a finite part. The integrand introduced in (44) is defined by

$$\begin{aligned} I(z, \mathbf{l}) &:= \frac{C_A}{\pi} zP(z) \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \frac{\mathcal{N}C_A}{\mathbf{l}^2 + (\mathbf{l} - \mathbf{k})^2} \\ &\times \int dx \mathcal{S}_J^{(3)}(\mathbf{k} - z\mathbf{l}, z\mathbf{l}, xz; x) f_g^{(0)}(x). \end{aligned} \quad (45)$$

The soft term in (44) is defined by evaluating the integrand in the soft limit $z \rightarrow 0$. In this limit, the jet distribution can be simplified by means of (28b), and leads to the constraint $\mathbf{k}^2 = E_J^2$. One obtains

$$\begin{aligned} \frac{d\sigma_A^{f,\text{soft}}}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \int \frac{d\mathbf{l}}{\pi_\varepsilon(\mathbf{l} - \mathbf{k})^2} I(0, \mathbf{l}) \\ &= \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \int \frac{d\mathbf{l}}{\pi_\varepsilon(\mathbf{l} - \mathbf{k})^2} \\ &\times \frac{\mathcal{N}C_A}{\mathbf{l}^2 + (\mathbf{l} - \mathbf{k})^2} \int dx \mathcal{S}_J^{(2)}(\mathbf{k}; x) f_g^{(0)}(x) \end{aligned}$$

² This term has the same structure of the C_F term analyzed in [1], and can be simply recovered from the latter by replacing $C_F^2 \mathcal{P}_{gq} \rightarrow C_A^2 P$. The analysis that we perform here closely follows the one made in that paper

$$\begin{aligned} &= \alpha_s \frac{C_A}{\pi} \left[\frac{1}{2\varepsilon^2} - \frac{\pi^2}{12} \right] \left(\frac{E_J^2}{\mu^2} \right)^\varepsilon \\ &\times \int d\mathbf{k} \int dx h_q^{(0)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x), \end{aligned} \quad (46)$$

where as usual, in the final result, we have collected some factors in such a way that they reproduce the LO jet vertex (7). The divergent factor exhibits single as well as double poles, because our definition of the soft part includes also the region where collinear and soft singularities merge.

The pure collinear singularity can be isolated by evaluating the integrand (45) in the collinear limit $\mathbf{l} = \mathbf{k}$, after having subtracted the soft term ($\mathbf{l} = \mathbf{k}, z = 0$). The resulting expression is clearly regular in the soft limit ($z \rightarrow 0$) and therefore contains a simple collinear pole. An UV cut-off Λ is introduced since the residue at the collinear limit is no more integrable in the UV region. Thanks to (28c) the jet distribution simplifies to $\mathcal{S}_J^{(2)}$:

$$\begin{aligned} \frac{d\sigma_A^{f,\text{coll}}}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \int \frac{d\mathbf{l}}{\pi_\varepsilon(\mathbf{l} - \mathbf{k})^2} \\ &\times [I(z, \mathbf{k}) - I(0, \mathbf{k})] \Theta(\Lambda^2 - (\mathbf{l} - \mathbf{k})^2) \\ &= \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \frac{\mathcal{N}C_A}{\mathbf{k}^2} \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \\ &\times [zP(z) - 1] \int \frac{d\mathbf{l}}{\pi_\varepsilon(\mathbf{l} - \mathbf{k})^2} \Theta(\Lambda^2 - (\mathbf{l} - \mathbf{k})^2) \\ &\times \int dx \mathcal{S}_J^{(2)}(\mathbf{k}; x) f_g^{(0)}(x) \\ &= \alpha_s \frac{C_A}{\pi} \left[-\frac{11}{12\varepsilon} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon + \frac{67}{36} \right] \\ &\times \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x). \end{aligned} \quad (47)$$

The remaining part is regular in the $\varepsilon \rightarrow 0$ limit and defines the finite term:

$$\begin{aligned} \frac{d\sigma_A^{f,\text{finite}}}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{l}}{\pi(\mathbf{l} - \mathbf{k})^2} [I(z, \mathbf{l}) - I(0, \mathbf{l}) \\ &- (I(z, \mathbf{k}) - I(0, \mathbf{k})) \Theta(\Lambda^2 - (\mathbf{l} - \mathbf{k})^2)] \\ &= \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int_0^1 \frac{dz}{(1-z)_+} \\ &\times [(1-z)P(1-z)] \int dx f_g^{(0)}(x) \\ &\times \int \frac{d\mathbf{l}}{\pi\mathbf{l}^2} \left[\frac{\mathcal{N}C_A}{\mathbf{l}^2 + (\mathbf{l} - \mathbf{k})^2} \right. \\ &\times \mathcal{S}_J^{(3)}(\mathbf{l} + z(\mathbf{k} - \mathbf{l}), (1-z)(\mathbf{k} - \mathbf{l}), x(1-z); x) \\ &\left. - V_g^{(0)}(\mathbf{k}, x) \Theta(\Lambda^2 - \mathbf{l}^2) \right] \end{aligned} \quad (48)$$

In the last equation we have performed the change of variable $z \rightarrow 1 - z$ in order to simplify the expression with the introduction of the $(\)_+$ regularization for regularizing the $1/(1-z)$ distribution at $z = 1$.

Next we consider the term with the **initial state** (i) collinear singularity. We can write, in the same way as before,

$$\begin{aligned} \frac{d\sigma^i}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \int \frac{d\mathbf{l}}{\pi_\varepsilon \mathbf{l}^2} I(z, \mathbf{l}) \\ &= \frac{d\sigma_A^{i,\text{soft}}}{dJ} + \frac{d\sigma_A^{i,\text{coll}}}{dJ} + \frac{d\sigma_A^{i,\text{finite}}}{dJ}, \end{aligned} \quad (49)$$

where I is given by (45). It is trivial to see that the soft contribution, defined by evaluating the integrand I in (49) at $z = 0$, is equal to the corresponding (f) term (46):

$$\frac{d\sigma_A^{i,\text{soft}}}{dJ} = \frac{d\sigma_A^{f,\text{soft}}}{dJ}. \quad (50)$$

As to the collinear piece, we note that in the collinear limit $\mathbf{l} = 0$ the jet distribution reduces (by applying (28e)) to $\mathcal{S}_J^{(2)}$, and one gets, using always the same UV cutoff Λ , performing the \mathbf{l} integration and changing the integration variable $z \rightarrow 1 - z$,

$$\begin{aligned} \frac{d\sigma_A^{i,\text{coll}}}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z^{1-2\varepsilon}} \int \frac{d\mathbf{l}}{\pi_\varepsilon \mathbf{l}^2} \\ &\quad \times [I(z, \mathbf{0}) - I(0, \mathbf{0})] \Theta(\Lambda^2 - \mathbf{l}^2) \\ &= \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int dx f_g^{(0)}(x) \\ &\quad \times \left\{ \frac{1}{\varepsilon} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon \int_0^1 \frac{dz}{(1-z)_+} (1-z) P(1-z) \right. \\ &\quad \times V_g^{(0)}(\mathbf{k}, xz) + 2 \int_0^1 dz \left[\frac{\ln(1-z)}{(1-z)} \right]_+ \\ &\quad \left. \times (1-z) P(1-z) V_g^{(0)}(\mathbf{k}, xz) \right\}. \end{aligned} \quad (51)$$

The left contribution is regular in 4 dimensions and defines another finite term

$$\begin{aligned} \frac{d\sigma_A^{i,\text{finite}}}{dJ} &:= \alpha_s \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{l}}{\pi \mathbf{l}^2} [I(z, \mathbf{l}) - I(0, \mathbf{l}) \\ &\quad - (I(z, \mathbf{0}) - I(0, \mathbf{0})) \Theta(\Lambda^2 - \mathbf{l}^2)] \\ &= \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int_0^1 \frac{dz}{(1-z)_+} \\ &\quad \times [(1-z) P(1-z)] \int dx f_g^{(0)}(x) \\ &\quad \times \int \frac{d\mathbf{l}}{\pi \mathbf{l}^2} \left[\frac{\mathcal{N}C_A}{\mathbf{l}^2 + (\mathbf{l} - \mathbf{k})^2} \right. \\ &\quad \times \mathcal{S}_J^{(3)}(\mathbf{k} - (1-z)\mathbf{l}, (1-z)\mathbf{l}, x(1-z); x) \\ &\quad \left. - V_g^{(0)}(\mathbf{k}, xz) \Theta(\Lambda^2 - \mathbf{l}^2) \right]. \end{aligned} \quad (52)$$

4.3.2 B term

The last piece to be analyzed is the B term, which reads

$$\frac{d\sigma_B}{dJ} = \alpha_s \frac{C_A}{\pi} \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int dx f_g^{(0)}(x)$$

$$\begin{aligned} &\times \int_{z_{\text{cut}}}^1 dz (1-z) P(z) \int \frac{d\mathbf{k}'}{\pi_\varepsilon \mathbf{k}'^2} \\ &\times \frac{\mathbf{q} \cdot (\mathbf{q} - z\mathbf{k})}{\mathbf{q}^2 (\mathbf{q} - z\mathbf{k})^2} \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) \\ &= \frac{d\sigma_B^{\text{coll}}}{dJ} + \frac{d\sigma_B^{\text{LL}}}{dJ} + \frac{d\sigma_B^{\text{const}}}{dJ}, \end{aligned} \quad (53)$$

and will be decomposed in a collinear divergent piece, an energy dependent piece which is just the LL contribution to the cross section coming from gluon emission in the central region, plus a finite and constant in energy term as shown in the last line of the above equation. It can be noticed immediately the presence of a $\mathbf{g} \parallel 1$ collinear singularity corresponding to the $\mathbf{k}' = 0$ pole. As already commented for the similar case [1] there are no other singularities in the \mathbf{k}' -integration (neither $\mathbf{q} = 0$ nor $\mathbf{q} - z\mathbf{k} = 0$) except for $z \rightarrow 0$ corresponding to gluon 3 being in the central region, which gives the LL contribution. Here we expect a soft singularity needed to cancel the one in the gluon trajectory (16).

We recall that the jet distribution functions become essential in disentangling the collinear singularities, the soft singularities, and the leading $\log s$ pieces.

The basic mechanism are the same as for the quark case (remember here that gluon 3 is playing a special role because of the manipulation in the integrand performed at the beginning of Sect. 4.3):

- When the outgoing gluon 1 is in the collinear region of the incoming gluon \mathbf{g} , i.e., $y_1 \rightarrow \infty$, it cannot enter the jet; only gluon 3 can thus be the jet, y_3 is fixed and no logarithm of the energy can arise due to the lack of evolution in the gluon rapidity. No other singular configuration is found when $J = \{3\}$.
- In the composite jet configuration, i.e., $J = \{1, 3\}$, the gluon rapidity is bounded within a small range of values, and also in this case no $\log s$ can arise. There could be a singularity for vanishing gluon 3 momentum: even if the $1 \parallel 3$ collinear singularity is absent, we have seen that, at very low z , a soft singular integrand arises. However, the divergence is prevented by the jet cone boundary, which causes a shrinkage of the domain of integration $\sim z^2$ for $z \rightarrow 0$ and thus compensates the growth of the integrand.
- The jet configuration $J = \{1\}$ allows gluon 3 to span the whole phase space, apart, of course, from the jet region itself. The LL term arises from gluon configurations in the central region. Therefore, it is crucial to understand to what extent the differential cross section provides a leading contribution. It turns out that the coherence of QCD radiation suppresses the emission probability for gluon 3 rapidity y_3 being larger than the rapidity y_1 of the gluon 1, namely an angular ordering prescription holds. This will provide the final form of the leading term, i.e., the appropriate scale of the energy and, as a consequence, a finite and definite expression for the one-loop jet vertex correction.

Let us isolate in (53) the initial state $\mathbf{g} \parallel 1$ collinear singular contribution and define, as usual, the collinear

Table 1. Schematics of the decomposition of real and virtual one-loop corrections to gq scattering and references of the corresponding equations

virtual				real qgg								real $qq\bar{q}$				
$\omega^{(1)}$	$\tilde{\Pi}_g$	$\tilde{\Pi}_q$	$y'_3 < 0$	$y'_3 > 0$												
				A				B				C_F		C_A		
				f				i								
LL	soft coll	LL	$h_q^{(1)}$	soft	coll	fin	soft	coll	fin	coll	LL	const	coll	fin	coll	fin
(59)	(59)	(59)	(25)	(46)	(47)	(48)	(50)	(51)	(52)	(54)	(59)	(58)	(35)	(36)	(38)	(39)

term by setting $\mathbf{k}' = 0$ (except in the $1/\mathbf{k}'^2$ pole), and by introducing an UV cutoff. Observing that the jet distribution, because of (28d), reduces to $\mathcal{S}_J^{(2)}(\mathbf{k}; xz)$ one easily obtains

$$\frac{d\sigma_B^{\text{coll}}}{dJ} = \alpha_s \frac{C_A}{\pi} \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int_0^1 dz V_g^{(0)}(\mathbf{k}, xz) \times f_g^{(0)}(x) \left[\frac{1}{\varepsilon} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon P(z) \right]. \quad (54)$$

The LL part can be extracted exactly with the same procedure already followed in [1], thanks to the suppression of the partonic cross section when gluon 3 is emitted at larger rapidity w.r.t. that of gluon 1. Infact, when gluon 3 is in the central region, gluon 1 must be the jet. In this case, the azimuthal average of the cross section w.r.t. gluon 3 azimuthal angle ϕ_3 at fixed gluon 1 momentum yields

$$\left\langle \frac{(1-z)\mathbf{q} \cdot (\mathbf{q} - z\mathbf{k})}{q^2(\mathbf{q} - z\mathbf{k})^2} \right\rangle_{\phi_3} = \frac{1}{q^2} \Theta(E_3 - z(E_1 + E_3)). \quad (55)$$

This relation is exact and clearly shows that, outside the angular ordered region

$$\frac{E_3}{z} > \frac{E_1}{1-z} \iff \theta_3 > \theta_1 \iff y_3 < y_1, \quad (56)$$

there is no contribution to the cross section. In practice, by taking into account the variation of $h_q^{(0)}(\mathbf{k})$ during the averaging procedure, instead of a strictly vanishing contribution we have a strong suppression. Note that, in the limit $\mathbf{q} \rightarrow 0$ (which includes the soft region), the variation of \mathbf{k}' goes to zero as well, so that (56) is really an accurate statement in the “dangerous” part of the phase space. Moreover, (55) shows that the $1/q^2$ kinematic dependence of the LL kernel governs the differential cross section up to the very end of the angular boundary.

Therefore, we define the LL contribution in the “lower half region” $y'_3 > 0$ by

$$\frac{d\sigma_B^{\text{LL}}}{dJ} := \alpha_s \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) \int d\mathbf{k}' \frac{C_A}{\pi} \frac{1}{\pi_\varepsilon q^2} h_g^{(0)}(\mathbf{k}')$$

$$\begin{aligned} & \times \int_{z_{\text{cut}}}^1 \frac{dz}{z} \Theta(E_3 - z(E_1 + E_3)) \\ & \times \int dx \mathcal{S}_J^{(2)}(\mathbf{k}', x) f_g^{(0)}(x) \\ & = \int dx \int d\mathbf{k} \int d\mathbf{k}' h_q^{(0)}(\mathbf{k}) K^{(0,\text{real})}(\mathbf{k}, \mathbf{k}') \\ & \times \log \frac{\sqrt{xs}}{E_J + E_3} V_g^{(0)}(\mathbf{k}', x) f_g^{(0)}(x), \quad (57) \end{aligned}$$

having imposed the jet condition $J = \{1\}$.

The remaining part of the B term is finite in 4 dimensions and constant in energy, so that we can set $\varepsilon = 0$ and $z_{\text{cut}} = 0$ to define the constant part

$$\frac{d\sigma_B^{\text{const}}}{dJ} := \left[\frac{d\sigma_B}{dJ} - \frac{d\sigma_B^{\text{coll}}}{dJ} - \frac{d\sigma_B^{\text{LL}}}{dJ} \right]_{\substack{z_{\text{cut}}=0 \\ \varepsilon=0}}. \quad (58)$$

5 The NLO jet vertex: sum of real and virtual corrections

Having completed the calculation of both the virtual and real corrections in the whole phase space, we are going to collect all partial results and to show that the complete one-loop jet cross section can naturally be fitted to the form of (12). Table 1 summarizes the decomposition of the one-loop jet cross section and gives the references of the various contributions.

In Sect. 3 we have presented the virtual contributions to the jet cross section which, after renormalization of the coupling, assume the form of (15). The contribution coming from the $\tilde{\Pi}_q$ impact factor correction has been combined with the “upper half region” real contribution to give the full impact factor of the upper quark q (second term of (25)).

The remaining virtual terms and the LL contributions of both the negative (first term of (25)) and positive ((57)) rapidity regions can be conveniently rewritten in the form

$$\frac{d\sigma^{(\text{virt})}}{dJ} \Big|_{\omega^{(1)}} + \frac{d\sigma^{(\text{virt})}}{dJ} \Big|_{\tilde{\Pi}_g} + \frac{d\sigma^{(\text{real})}}{dJ} \Big|_{\text{LL}} =$$

$$\begin{aligned}
&= \alpha_s \int dx \int d\mathbf{k} d\mathbf{k}' h_q^{(0)}(\mathbf{k}) K^{(0)}(\mathbf{k}, \mathbf{k}') \\
&\quad \times \log \frac{xs}{s_0(\mathbf{k}, \mathbf{k}')} V_g^{(0)}(\mathbf{k}', x) f_g^{(0)}(x) \\
&\quad + \alpha_s \left[\left(\frac{\mathbf{k}^2}{\mu^2} \right)^\varepsilon \left(-\frac{C_A}{\pi} \frac{1}{\varepsilon^2} + \frac{11C_A - 2N_f}{6\pi} \frac{1}{\varepsilon} \right) \right. \\
&\quad \left. + \frac{C_A}{\pi} \left(\frac{5}{12} \pi^2 - \frac{67}{36} \right) + \frac{5}{18} \frac{N_f}{\pi} - b_0 \log \frac{\mathbf{k}^2}{\mu^2} \right] \\
&\quad \times \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) V_g^{(0)}(\mathbf{k}, x) f_g^{(0)}(x), \quad (59)
\end{aligned}$$

where the kernel $K^{(0)}$ is the full BFKL kernel and the energy scale s_0 in (59) has been chosen as

$$s_0(\mathbf{k}, \mathbf{k}') := (|\mathbf{k}'| + |\mathbf{q}|)(|\mathbf{k}| + |\mathbf{q}|) = (E_J + E_3)(E_2 + E_3). \quad (60)$$

Let us stress that the energy scale in (60) arises naturally when one requires impact factors and PDFs to have standard collinear properties and the remaining non-leading-log term to be finite in both the physical $\varepsilon \rightarrow 0$ and high-energy $s \rightarrow \infty$ limits. Choosing a different scale of the energy requires the introduction of additional NLL operators, which has to be added as multiplicative corrections to the Green's function. If, for instance, we adopt $s_0 = |\mathbf{k}||\mathbf{k}'|$, then the Green's function (10b) has to be replaced by

$$G(xs, \mathbf{k}, \mathbf{k}') = (1 + \alpha_s H_L) \left[1 + \alpha_s K^{(0)} \log \frac{xs}{|\mathbf{k}||\mathbf{k}'|} \right] \times (1 + \alpha_s H_R) \quad (61)$$

$$\begin{aligned}
H_L(\mathbf{k}, \mathbf{k}') &= -K^{(0)}(\mathbf{k}, \mathbf{k}') \log \frac{|\mathbf{k}| + |\mathbf{q}|}{|\mathbf{k}|} \\
H_R(\mathbf{k}, \mathbf{k}') &= H_L(\mathbf{k}', \mathbf{k}). \quad (62)
\end{aligned}$$

We now consider the sum of all ε -divergent contributions coming from both real and virtual corrections. All double poles coming from (59), (46) and (50) cancel out. The single poles come from (59), (35), (38), (47), (51), (54). They combine to give exactly the DGLAP splitting functions terms which define the PDF one-loop corrections. One has only to note that

$$\frac{(1-z)P(1-z)}{(1-z)_+} + P(z) = \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \quad (63)$$

and that the P_{gg} LO DGLAP splitting function is defined by

$$\begin{aligned}
P_{\text{gg}}(z) &= 2C_A \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) \right] \\
&\quad + \left(\frac{11C_A}{6} - \frac{N_f}{3} \right) \delta(1-z). \quad (64)
\end{aligned}$$

The non-LL total singular contribution adds up to

$$\frac{d\sigma^{\text{singular}}}{dJ} = \frac{\alpha_s}{2\pi} \frac{1}{\varepsilon} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k})$$

$$\begin{aligned}
&\times \left\{ V_q^{(0)}(\mathbf{k}, x) [2N_f P_{\text{qg}} \otimes f_g^{(0)}](x) \right. \\
&\quad \left. + V_g^{(0)}(\mathbf{k}, x) [P_{\text{gg}} \otimes f_g^{(0)}](x) \right\}. \quad (65)
\end{aligned}$$

The cutoff Λ introduced in all the collinear subtraction has the physical interpretation of factorization scale and can therefore be identified with μ_F . Equation (65) shows that the only singular terms remaining after the sum of real and virtual corrections are of collinear origin, and actually amount to the expected collinear singularities stemming from parton radiation out of incoming gluons. They can be absorbed in the PDFs according to (13); of course only the term $\mathbf{b} = \mathbf{g}$ in the sum is reproduced.

Finally all the finite and not LL terms from (59), (35), (36), (38), (39), (46), (50), (47), (51), (48), (52) and (58) contribute to the jet vertex

$$\frac{d\sigma^{\text{finite}}}{dJ} = \alpha_s \int dx \int d\mathbf{k} h_q^{(0)}(\mathbf{k}) V_g^{(1)}(\mathbf{k}, x) f_g^{(0)}(x), \quad (66)$$

which defines the NLO correction to the gluon-initiated jet vertex

$$\begin{aligned}
V_g^{(1)}(\mathbf{k}, x) &:= \left[\left(\frac{11}{6} \frac{C_A}{\pi} - \frac{1}{3} \frac{N_f}{\pi} \right) \log \frac{\mathbf{k}^2}{\Lambda^2} + \frac{\pi^2 C_A}{4\pi} + \frac{13}{36} \frac{N_f}{\pi} \right. \\
&\quad \left. - b_0 \log \frac{\mathbf{k}^2}{\mu^2} \right] V_g^{(0)}(\mathbf{k}, x) \\
&\quad + \int dz V_g^{(0)}(\mathbf{k}, xz) \left[\frac{N_f}{\pi} \frac{C_F}{C_A} z(1-z) \right. \\
&\quad \left. + 2 \frac{C_A}{\pi} (1-z) P(1-z) \left(\frac{\log(1-z)}{1-z} \right)_+ \right] \\
&\quad + \frac{N_f}{\pi} \int \frac{d\mathbf{k}'}{\pi} \int_0^1 dz P_{\text{qg}}(z) \left[\frac{h_q^{(0)}(\mathbf{k}')}{\mathbf{q}^2 + \mathbf{k}'^2} \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) \right. \\
&\quad \left. - \frac{1}{\mathbf{k}'^2} \Theta(\Lambda^2 - \mathbf{k}'^2) V_q^{(0)}(\mathbf{k}, xz) \right] \\
&\quad + \frac{N_f}{2\pi} \int \frac{d\mathbf{k}'}{\pi} \int_0^1 dz P_{\text{qg}}(z) \frac{\mathcal{N}C_A}{(\mathbf{q} - z\mathbf{k})^2} \\
&\quad \times \left[z(1-z) \frac{\mathbf{q} \cdot \mathbf{k}'}{\mathbf{q}^2 \mathbf{k}'^2} \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) \right. \\
&\quad \left. - \frac{1}{\mathbf{k}^2} \Theta(\Lambda^2 - (\mathbf{q} - z\mathbf{k})^2) \mathcal{S}_J^{(2)}(\mathbf{k}, x) \right] \\
&\quad + \frac{C_A}{\pi} \int_0^1 \frac{dz}{(1-z)_+} [(1-z)P(1-z)] \\
&\quad \times \int \frac{d\mathbf{l}}{\pi \mathbf{l}^2} \left\{ \frac{\mathcal{N}C_A}{\mathbf{l}^2 + (\mathbf{l} - \mathbf{k})^2} \right. \\
&\quad \times \left[\mathcal{S}_J^{(3)}(z\mathbf{k} + (1-z)\mathbf{l}, (1-z)(\mathbf{k} - \mathbf{l}), x(1-z); x) \right. \\
&\quad \left. + \mathcal{S}_J^{(3)}(\mathbf{k} - (1-z)\mathbf{l}, (1-z)\mathbf{l}, x(1-z); x) \right] \\
&\quad \left. - \Theta(\Lambda^2 - \mathbf{l}^2) [V_g^{(0)}(\mathbf{k}, x) + V_g^{(0)}(\mathbf{k}, xz)] \right\} \\
&\quad + \frac{C_A}{\pi} \int \frac{d\mathbf{k}'}{\pi} \int dz \left[P(z) \left((1-z) \frac{\mathbf{q} \cdot (\mathbf{q} - z\mathbf{k})}{\mathbf{q}^2 (\mathbf{q} - z\mathbf{k})^2} \right) \right.
\end{aligned}$$

$$\begin{aligned}
 & \times h_g^{(0)}(\mathbf{k}') \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) + \\
 & - \frac{1}{\mathbf{k}'^2} \Theta(\Lambda^2 - \mathbf{k}'^2) V_g^{(0)}(\mathbf{k}, xz) \Big) \\
 & - \frac{1}{z\mathbf{q}^2} \Theta(|\mathbf{q}| - z(|\mathbf{q}| + |\mathbf{k}'|)) V_g^{(0)}(\mathbf{k}', x) \Big]. \quad (67)
 \end{aligned}$$

Like for the quark case the gluon originated vertex clearly depends on the jet definition and on three scales: the energy scale s_0 (via the subtraction of the LL term $\propto 1/z$), the factorization scale $\Lambda = \mu_F$ and the renormalization scale μ . Note that, both for the quark and for the gluon initiated vertex, the μ -dependence appears only inside the term $b_0 \log \mathbf{k}^2/\mu^2$ which multiplies the Born approximation. If we chose, for example, $\Lambda^2 = E_J^2$, then all the other terms in (67) are scale invariant.

6 A consistency check: jet vertex versus impact factor

In this section we compare the structures of the high energy factorization formulae in the two cases of jet cross section – discussed in this paper and in [1] – and of inclusive partonic cross section – presented in [13,14]. This is interesting both for understanding the different structure of the singularities involved in the two formulations and, on the other hand, to underline the equivalence in the LL subtraction procedure which will allow us to use the results of two-loop calculations for the generalization to higher orders presented in Sect. 7.

6.1 Jet cross section versus inclusive cross section

Let us consider the quark-hadron one-jet inclusive factorization formula

$$\frac{d\sigma_{qH}}{dJ} = \sum_a \int dx d\mathbf{k} d\mathbf{k}' h_q(\mathbf{k}) G(xs, \mathbf{k}, \mathbf{k}') V_a(\mathbf{k}', x) f_a(x) \quad (68)$$

(cfr. (10a)) and compare it to the one describing the inclusive quark-parton cross section

$$d\sigma_{qa}^{\text{incl}} = \int d\mathbf{k} d\mathbf{k}' h_q(\mathbf{k}) G(\hat{s}, \mathbf{k}, \mathbf{k}') h_a(\mathbf{k}'). \quad (69)$$

Formally, one can obtain the former from the latter by simply replacing the partonic impact factor of the lower incoming parton \mathbf{a} with the product of jet vertex and PDF – we refer to this product as “jet impact factor”. There is nevertheless an important difference between the one-loop correction to the impact factor $h^{(1)}$ and the one to the “jet impact factor” $[V * f]^{(1)} = V^{(1)} * f^{(0)} + V^{(0)} * f^{(1)}$. In the gluon-initiated process, the jet impact factor contains the full collinear singularities (included in $f^{(1)}$) relative to the $\mathbf{g} \rightarrow \mathbf{q}$ and $\mathbf{g} \rightarrow \mathbf{g}$ splittings, as shown in (65). The gluon impact factor, on the other hand, contains the full $\mathbf{g} \rightarrow \mathbf{q}$ collinear singularity but only a part of the $\mathbf{g} \rightarrow \mathbf{g}$ one. In fact, the residue of the $1/\varepsilon$ pole of the gluon impact

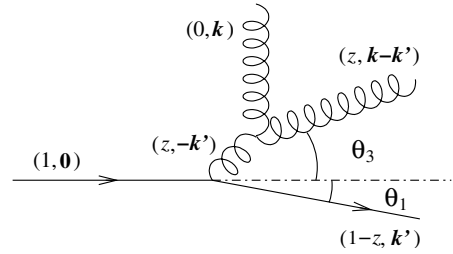


Fig. 5. LAB frame view of the *quark + reggeon* \rightarrow *quark + gluon blob* involved in the quark-initiated jet cross section (in particular the Feynman diagram responsible for the $1 \parallel \mathbf{a}$ collinear singularity yielding $1/\varepsilon \times P_{gq}$ is shown). θ_1 and θ_3 denotes the outgoing quark and gluon polar angles respectively. In parentheses close to each particles are the longitudinal momentum fraction (w.r.t. the incoming parton) and the transverse momentum

factor correction $h_g^{(1)}$ is $\int dz [P_{qg}(z) + P_{gg}(z) - 2C_A/z]$. This is because the subtracted term $2C_A/z$ has been taken into account in the LL contribution (see (57)). The same is true for quark-initiated processes, the residue of the $1/\varepsilon$ pole in the quark impact factor correction $h_q^{(1)}$ being $\int dz [P_{qq}(z) + P_{gq}(z) - 2C_F/z]$.

In the jet case, it has been possible to factorize the full collinear singularities into the PDF because of the jet distribution, which selects only some final states out of the whole phase space.

How does this happen? Let us discuss this issue in more detail for the simpler case of quark-initiated scattering. Figure 5 shows a possible configuration of the quark-reggeon blob $qg^* \rightarrow qg$. In the inclusive cross section, all configurations contribute to the cross section. When $\theta_3 < \theta_1$, the contribution is entirely assigned to the impact factor correction. However, when $\theta_3 > \theta_1$, according to angular ordering, the part $\propto 2C_F/z$ of the squared matrix element is interpreted as LL term. Since the collinear region which should provide the P_{gq} splitting function times the $1/\varepsilon$ pole is just $\theta_1 \simeq 0$, it is clear that it is always affected by the LL subtraction, and therefore the impact factor correction contains only the “non singular” part $P_{gq}(z) - 2C_F/z$ of the splitting function.

In the jet case, in the collinear region $\theta_1 \simeq 0$, we perform no LL subtraction, because the gluon is constrained to be the jet, hence the $1/\varepsilon$ collinear pole multiplies the full P_{gq} splitting function (and the LO jet vertex). The LL subtraction is performed when $\theta_3 > \theta_1$ and the jet consists of only the quark. In this case there is no collinear singularity associated with the LL subtraction.

6.2 Relation between $h^{(1)}$ and $V^{(1)}$

One could think to check the consistency of our calculation by integrating the factorization formula of the jet observables with respect to the jet variables in order to recover the quark-hadron inclusive cross section

$$d\sigma_{qH}^{\text{incl}} = \sum_a \int dx d\mathbf{k} d\mathbf{k}' h_q(\mathbf{k}) G(xs, \mathbf{k}, \mathbf{k}') h_a(\mathbf{k}') f_a^{(0)}(x), \quad (70)$$

obtained by convoluting the quark-parton inclusive cross section (69) with the bare³ parton densities $f_a^{(0)}(x)$.

This is not the case, because the single-jet configurations would be counted twice. Consider, for instance, the configuration of Fig. 5. It would be counted once when the quark is the jet and a second time when the gluon is the jet. On the contrary, if we replace the jet distribution $\mathcal{S}_J^{(3)}$ with another distribution $\tilde{\mathcal{S}}_J^{(3)}$ with the property that

$$\int dJ \tilde{\mathcal{S}}_J^{(3)}(\mathbf{p}_1, \mathbf{p}_3, \xi; x) = 1, \quad dJ := dy_J dE_J d\phi_J, \quad (71)$$

(i.e., a function that counts each final state once and only once), and define a ‘‘modified jet vertex’’ $\tilde{V}^{(1)}$, obtained from $V^{(1)}$ by replacing $\mathcal{S}_J^{(3)} \rightarrow \tilde{\mathcal{S}}_J^{(3)}$, then the integration over the jet variables of the ‘‘modified one-jet inclusive cross section’’ yields the inclusive cross section (70), because

$$\int dJ \tilde{V} * f = h * f^{(0)}. \quad (72)$$

More precisely:

$$\begin{aligned} & \int dJ \left\{ \tilde{V}_a^{(1)}(\mathbf{k}, x; J) + \frac{1}{2\pi\varepsilon} \left(\frac{\Lambda^2}{\mu^2} \right)^\varepsilon \right. \\ & \quad \times \left. \int dz [V_a^{(0)}(\mathbf{k}, xz; J) P_{aq}(z) + V_a^{(0)}(\mathbf{k}, xz; J) P_{ag}(z)] \right\} \\ & = h_a^{(1)}(\mathbf{k}), \quad (a = q, g). \end{aligned} \quad (73)$$

Despite of the difference between the partonic and jet impact factors, the remaining factors in the respective factorization formulae are the same. In particular, the Green’s function at scale $|\mathbf{k}||\mathbf{k}'|$ in the form of (61) is identical to the one defined in [13] for the partonic case, i.e., it shares the same impact kernel H_L and H_R .

This is due to the method adopted for the definition of the partonic impact factor (resp., the jet vertex), i.e., for the separation of the central region from the fragmentation region. In practice, whatever the selection of the final state for the observable under study, the precise contribution of the fragmentation region of each of the incoming particles has been defined according to the following scheme:

- take the expression of the squared matrix element (supplied eventually with the distribution \mathcal{S} providing the final state selection) valid when the gluon is in the fragmentation region of the incoming particle;
- extract the central region limit from the above expression. It should be proportional to the BFKL kernel times the LO impact factor times an enhancing factor describing the large extension of the central region (in our variables, this enhancing factor is $1/z$), provided the final state selection is inclusive in the central region. This is the case for the one- and two-jet inclusive observables that we are studying;

- at *fixed quark momentum*, subtract from the fragmentation region expression the one obtained as central region limit. This subtraction has to be performed according to the angular ordering prescription $\theta_3 > \theta_1 \iff y_3 > y_1$ in the soft gluon region.

This defines on one side the impact factors, and on the other side what is not impact factor, i.e., what contributes to the Green’s function. The universality of the Green’s function (at least at one-loop) should now become clearer: it is due to the same scheme of defining the LL subtraction, yielding in all cases the BFKL kernel times the LO impact factor. The details of the angular ordering out of the soft gluon region and the choice of the energy scale determines the form of the impact kernel $H_{L,R}$.

7 Factorization formula at higher orders

In this section we discuss some issues related to the generalization of the factorization formula (10) to higher order perturbation theory.

It was the main purpose of this paper (and of the previous one [1]) to provide a factorization formula for the jet production cross section at high energies, accurate in the next-to-leading log s approximation. To this end we have studied the jet cross section formula in next-to-leading order, $\mathcal{O}(\alpha_s^3)$. The leading logarithmic approximation (LL) retains the coefficient of $\ln s$. What we have computed in this paper is the next-to-leading logarithmic approximation (NLL), i.e. the terms without the $\ln s$ enhancement. The generalization to higher order in α_s therefore contains, in LL, terms of the form $\alpha_s^n (\ln s)^{n-2}$, and in NLL contributions of the form $\alpha_s^n (\ln s)^{n-3}$. We are interested in summing both the LL and NLL terms to all orders α_s .

In generalizing our result to all orders we have to rely on Regge factorization properties of scattering amplitudes in the high energy limit. Regge theory states that amplitudes (both for elastic scattering, and also for the inelastic production of particles in the multiregge limit, when described by the exchange of Regge poles) obey factorization properties. In QCD the gluon has been found to reggeize, and, as it is discussed in some detail in the original BFKL papers [4] and in the NLO calculations of the BFKL kernel [8,9], scattering amplitudes with color octet exchange are expected to satisfy these factorization properties. In QCD calculations, this leads to the ‘Regge ansatz’ for elastic parton-parton scattering and for inelastic gluon production in the the multiregge limit. Strictly speaking, this ansatz is a hypothesis, suggested by Regge theory and by the observation that the gluon reggeizes. The strongest support for the correctness of the assumption comes from the result that the LL BFKL equation, derived from the Regge ansatz, in the color octet channel satisfies the bootstrap condition. More recently, the Regge ansatz has been used also in NLO calculations [8–10]; the bootstrap condition in the NLO BFKL equation was first formulated in its weak form inside elastic scattering amplitudes and after in its strong form, which is the one necessary for the self-consistency of the assumption of Reggeized form of

³ One must use the bare PDFs because the collinear singularities are embodied in the partonic impact factors

the production amplitudes [17, 18]. Very recently the validity of the bootstrap condition (strong form) in the NLO BFKL equation has been proven in [19].

In our case, we have used the Regge ansatz for the factorizing form in (10): in the jet production amplitude the large rapidity gap between the outgoing quark (at the upper end) and the jet (at the bottom) leads to the exchange of a reggeized gluon, and the upper and lower vertex become independent from each other. In the cross section formula (10) this factorization property leads to the impact factor and the jet vertex. In our calculation, it has been one of our tasks to show that the jet vertex fits into this factorization scheme: when integrated over the jet variables, the square of the “modified jet vertex” turns into the gluon (or quark) impact factor which has been studied before. Without the integration over the jet variables we have, in principle, a new situation: we find that Regge factorization still works at the NLO level. One expects that this factorization continues to hold also in higher order α_s , i.e. the impact factor at the upper end of the gluon and the jet vertex allow for a systematic expansion in powers of α_s . Similarly, the exchanged gluon which in lowest order is elementary turns into a reggeized gluon with a trajectory function being given as a power series in α_s .

When generalizing to higher orders, i.e. to the production of more gluons, we follow the strategy pursued in the NLO calculations of the BFKL kernel: the Regge ansatz for multi-gluon production amplitudes states that vertices that are separated by large rapidity gaps are independent of each other, i.e. they factorize. Before turning to the all-order formula, it may be instructive to take a closer look into the next order correction to the cross section formula, the two-loop jet cross section $\mathcal{O}(\alpha_s^4)$.

Since final states may consist of four, three, or two partons we have terms of order $\alpha_s^4 \log^2 s$ and $\alpha_s^4 \log s$ (terms without $\ln s$ belong to NNLL and will not be considered). Regge factorization leads us to expect:

$$\begin{aligned} \frac{1}{\alpha_s^4} \frac{d\sigma^{(2)}}{dJ} &= \frac{1}{2} \log^2 \frac{xs}{s_R} h^{(0)} K^{(0)} K^{(0)} V^{(0)} f^{(0)} \\ &+ \log \frac{xs}{s_R} \left[h^{(1)} K^{(0)} V^{(0)} f^{(0)} \right. \\ &+ h^{(0)} (H_L K^{(0)} + K^{(0)} H_R + K^{(1)}) V^{(0)} f^{(0)} \\ &\left. + h^{(0)} K^{(0)} V^{(1)} f^{(0)} + h^{(0)} K^{(0)} V^{(0)} f^{(1)} \right]. \end{aligned} \quad (74)$$

The first term on the RHS is the LL term given by the iteration of the leading kernel $K^{(0)}$. The square brackets collect the NLL contributions: the impact factor correction, the contribution from the products of impact kernel and leading kernel, the NLL kernel, and finally the corrections to the jet vertex and to the PDF.

In the second step we relate the terms of (74) to the various final states and kinematic regions contributing to the $\mathcal{O}(\alpha_s^4)$ cross section. We have three kinds of final states: two-parton final states (with two-loop virtual corrections), three-parton final states (with one-loop virtual

corrections) and four-parton final states at tree level. Because of the reggeization of the gluon, there is no one-to-one correspondence between the number of rapidity gaps and the power of $\ln s$. Let us go through the final states in some detail.

Consider first the four-parton final state of Fig. 6c. It can be shown that, when the two central gluons are emitted at large subenergies $s_{ij} \gg t_j$, i.e., final state particles are separated by large rapidity gaps, the cross section is described by the iteration of two BFKL kernels (actually their real parts), one for each emitted gluon. Since the external partons are isolated in rapidity space, they are described by LO partonic impact factors in the inclusive process, and by the LO jet vertices in the jet process.

NLL contributions arise from quasi-multi-regge kinematics, where one (and only one) pair of particles is emitted at small subenergy (small rapidity gap). If the pair with small subenergy is formed by the two central gluons 3 and 4, there is no ambiguity to associate the corresponding contribution to the NLL BFKL kernel $K^{(1)}$. This is one of the so called “irreducible contribution” of [9], because it can directly be interpreted as contribution to $K^{(1)}$, regardless of the definition of the impact factors. Note that the rapidity intervals $y_1 - y_3$ and $y_4 - y_2$ must be large in order to ensure the contribution to be NLL. Therefore the external particles 1 and 2 are still described by LO impact factors or jet vertices.

When one of the central gluon approaches an external particle, say gluon 3 enters the fragmentation region of particle 1, s_{13} and $y_1 - y_3$ become small. In this case we have the contribution to the jet vertex and PDF corrections. The definition given in the one-loop situation still works here, always because of high energy factorization: since s_{34} and s_{42} are much larger than s_{13} , the emission of gluon 4 is described by the real part of the LL BFKL kernel, independently of what is going on in the upper or lower vertices. Therefore, we can apply the machinery of the one-loop calculation factoring out the $b \rightarrow 2 + 4$ subprocess (described by $h^{(0)} K_{\text{real}}^{(0)}$, and consider only the blob $a + g^*(k) \rightarrow 1 + 3$ as done before. What one obtains – after the subtraction of the central region contribution of gluon 3 according to the angular ordering prescription – is the sum of the one-loop (real) corrections to the vertex and to the PDF, the impact kernel H_R , and a residual contribution that can only be assigned to the NLL kernel $K^{(1)}$. This is one of the so called “reducible” contribution, in that it depends on the definition of the “(jet) impact factors”. The key point is that also the reducible contributions to $K^{(1)}$ are the same as those obtained in the inclusive case, because they are related (just like H_R) to the subtraction of the central region from the fragmentation region, and we have shown in Sect. 6 that the LL subtraction is defined exactly in the same way in the jet cross section as well as in the inclusive cross section.

The treatment of the phase space region where gluon 4 is close to parton 2 is equivalent to that adopted in the inclusive process and provides the upper impact factor correction $h^{(1)}$, the other impact kernel H_L and another reducible contribution to $K^{(1)}$.

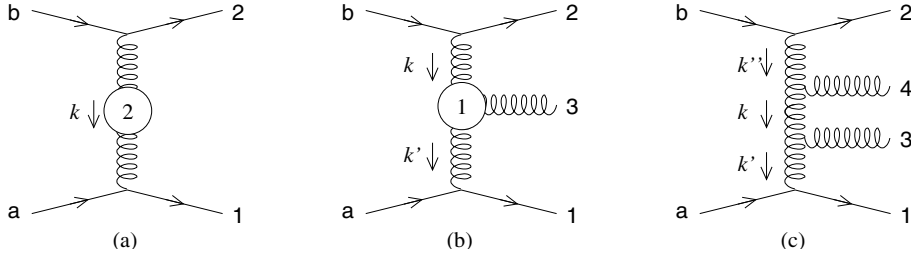


Fig. 6a–c. Schematics of amplitude diagrams involved in the calculation of the partonic cross section at relative $\mathcal{O}(\alpha_s^2)$: **a** two particle final state with two-loop virtual corrections; **b** three particle final state with one-loop virtual corrections; **c** four particle final state at tree level. The final state particles are ordered with increasing rapidity from the top to the bottom

A similar analysis can be performed for the three parton final state emission, Fig. 6b. When the gluon is in the central region, one has the product of the real part $K_{\text{real}}^{(0)}$ of the kernel (gluon emission) and of the virtual part $K_{\text{virt}}^{(0)}$ (provided by the leading virtual corrections), but also virtual corrections to the partonic or jet impact factors (given by virtual corrections attaching only to the corresponding partonic line) and an irreducible contribution to $K^{(1)}$ (given by virtual corrections attaching to the outgoing central gluon and to the exchanged gluons). When the emitted gluon approaches the lower partonic line, the contribution to the cross section can be at most NLL, and this occurs only if there are virtual corrections “filling” the large rapidity gap $y_3 - y_2$ providing a factorized one-loop gluon trajectory (virtual part of the BFKL kernel). Therefore, also in this case we can factor out the upper blob $b \rightarrow 2$ (described by $h^{(0)}K_{\text{virt}}^{(0)}$ and apply the procedure of separation of central region and fragmentation region to the lower blob $a + g^*(k) \rightarrow 1 + 3$ as done before. We obtain again – after the subtraction of the central region contribution of gluon 3 according to the angular ordering prescription – the sum of the one-loop (real) corrections to the vertex and to the PDF, the impact kernel H_R , and another reducible contribution to $K^{(1)}$.

The emitted gluon approaching the upper partonic line is treated in the same way of the inclusive case, giving again a correction to the upper partonic impact factor, the composition of the impact kernel H_L with the virtual part of the BFKL kernel and yet another reducible contribution to $K^{(1)}$.

The two-parton final state contribution (see Fig. 6a) is the simplest to analyze, since the jet has a trivial structure, namely it consists only of the lower outgoing quark, and is described by the LO jet vertex $V^{(0)} \propto h^{(0)}$. The comparison with the partonic case is therefore straightforward and we obtain: the iteration of two gluon trajectories $\omega^{(1)}$ (to be included in $K^{(0)}K^{(0)}$), the product of the virtual correction to the impact factor and the gluon trajectory (to be included in $h^{(1)}K^{(0)}$), the product of the trajectory and the virtual corrections to the lower quark-reggeon vertex (to be included in $K^{(0)}V^{(1)}$ and $K^{(0)}f^{(1)}$) and the two-loop gluon trajectory $\omega^{(2)}$ (the last irreducible contribution to $K^{(1)}$).

From (74) and from the subsequent discussion one sees that the quantity governing the high energy behaviour of the higher order processes at NLL level is the BFKL kernel

\mathcal{K} , which has the perturbative expansion

$$\mathcal{K} = \alpha_s K^{(0)} + \alpha_s^2 K^{(1)}. \quad (75)$$

Its NLL term $K^{(1)}$ has been determined for inclusive partonic scattering. In a perturbative expansion of the cross section, the NLL kernel $K^{(1)}$ enters the first time at relative order α_s^2 , i.e., at two-loop.

The most straightforward way of generalizing to all orders in α_s is to adapt the strategy used in the NLO study of parton parton scattering to our case of one-jet-inclusive processes. The main difference between the two cases is that in the latter the impact factor of the lower incoming parton is replaced by the convolution of the PDF with the jet vertex, as was already shown in the one-loop calculation. Therefore, we keep the general structure given in (10a), together with the perturbative expansions (11) of the energy independent factors. Only the Green’s function G needs to be extended to NLL level.

Before we write down the general all-order formula, let us return, once more, to the question of the energy scale s_0 . For future applications it will be convenient to use the *symmetric Regge-type energy scale* $s_R := |\mathbf{k}_1||\mathbf{k}_2|$. As we have discussed before, our requirement of giving the impact factor and the PDF’s the correct collinear properties has led to the energy scale (60), and a change of the energy scale will be induced by the additional operators $(1 + \alpha_s H_R)$, $(1 + \alpha_s H_L)$. As discussed in [13], this leads to the following form of the NLO BFKL Green’s function for the energy scale $s_R := |\mathbf{k}_1||\mathbf{k}_2|$:

$$G(xs, \mathbf{k}_1, \mathbf{k}_2) = \int \frac{d\omega}{2\pi i} \left(\frac{xs}{s_R} \right)^\omega \times \langle \mathbf{k}_1 | (1 + \alpha_s H_L) [\omega - \mathcal{K}]^{-1} (1 + \alpha_s H_R) | \mathbf{k}_2 \rangle. \quad (76)$$

in terms of the impact kernels (62) and of the BFKL kernel (75). Inserting this Green’s function into (10a) and expanding in powers of α_s one readily reproduces (74).

We conclude with a final formula for the Mueller-Navelet jets:

$$\frac{d^2\sigma}{dJ_1 dJ_2} = \sum_{a,b} \int dx_1 dx_2 \int d\mathbf{k}_1 d\mathbf{k}_2 f_a(x_1) V_a(\mathbf{k}_1, x_1) G(x_1 x_2 s, \mathbf{k}_1, \mathbf{k}_2) V_b(\mathbf{k}_2, x_2) f_b(x_2) \quad (77)$$

where $a, b = q, g$, and the subscripts 1 and 2 refer to jet 1 and 2 in hadron 1 and 2, resp. All elements are known in LO and in NLO; in particular, the jet vertex expressions $V_g^{(1)}$ from (67), and $V_q^{(1)}$ from (105) of [1].

8 Concluding remarks

In this paper we have completed the analytic part of the NLO corrections of the jet vertex which appears in the cross section formulae for Mueller-Navelet jets at hadron hadron colliders and for forward jets in deep inelastic electron proton scattering. The final result of our study is summarized in (77), the analytic expression of the cross section for the production of Mueller-Navelet jets at the Tevatron or at the LHC.

Apart from the interest in performing a consistent NLO analysis of the BFKL Pomeron at hadron hadron colliders or in deep inelastic ep scattering, the results of our analysis may raise some theoretical interest. Due to the very special kinematics of the jet production processes, the jet vertex lies at the interface between two different high energy limits, the hard scattering regime and the Regge limit (small- x limit): a priori it was not clear whether, at NLO accuracy, it would be possible to separate the collinear divergences from the BFKL-type gluon production. We have obtained an affirmative answer. Previous experience with partonic impact factors has served as a valuable guide in performing this separation inside the jet vertex.

What remains is the numerical evaluation of the cross section formulae, using the results derived in this paper. As the first step, one has to specify the jet functions $S_J^{(i)}$, i.e. we have to decide on a jet algorithm. In addition, experimental cuts have to be formulated. We hope to be able to report first numerical results in the near future.

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